PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Current Sheets

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Current sheets

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Current sheets are believed to be of prime importance in the solar atmosphere. Low down they may form at supergranulation boundaries, whereas up in the corona they have been suggested as a prominence formation site. In addition, they may occur when rapidly emerging flux presses up against pre-existing magnetic fields: if rapid magnetic field annihilation and reconnection is then triggered, a surge or a flare may be produced.

Comments are given about three aspects of general current sheet theory. The position and shape of the current sheet which forms between two-dimensional dipole sources is calculated. The thermal instability which occurs when the length of the sheet exceeds a critical value is described. Finally, a simple model of magnetic field annihilation is presented.

1. Introduction

Current sheets may form in many parts of the solar atmosphere. They may be present at supergranulation boundaries; if small bundles of flux are carried continuously to the edge of a supergranule, a spicule could be formed in the manner proposed by Uchida (1969) when the bundles in contact at a particular part of the boundary happen to contain oppositely directed magnetic fields. In the corona, current sheets exist in open magnetic field regions, where they have been suggested as sites for prominence formation (Kuperus & Tandberg-Hanssen 1967; Kuperus & Raadu 1974; Raadu & Kuperus 1973). In active regions especially, they may be created where newly emerging magnetic flux presses up against the pre-existing magnetic field (Rust 1975; Zirin 1974).

This paper presents comments about three aspects of current sheet theory – the magnetic configuration which results from the formation of a current sheet, the occurrence of thermal instabilities and the process of magnetic field annihilation.

2. The magnetic configuration

Sweet (1958) considered what happens when one starts with the potential field due to two magnetic dipoles as shown in figure 1, in which N is a neutral point where the magnetic field becomes zero. For simplicity, following Green (1965), Priest & Raadu (1975) have assumed the field is two dimensional and the sources are line dipoles of moment $(2\pi D/\mu)\hat{\boldsymbol{x}}$ situated at points $(a_0, 0)$ and $(-a_0, 0)$ in the x–y plane. The x- and y-components of the field are conjugate harmonic functions which can be written in terms of the complex variable $z = x+\mathrm{i}y$ as

$$B_{0y}+\mathrm{i}B_{0x} = \mathrm{i}D\,((z+a_0)^{-2}+(z-a_0)^{-2}). \tag{1}$$

If the dipoles approach one another at a rate which is much greater than the diffusion speed but much smaller than the Alfvén speed, then the field will remain 'frozen' to the plasma everywhere and will, we assume, pass through a series of quasi-equilibrium states. A neutral current sheet stretching from $P(0, p)$ to $Q(0, q)$ with $p > q$, say, is formed when the distance between the dipoles is $2a$ $(< 2a_0)$, as shown in figure 2; the length $(p-q)$ of the sheet varies as the dipoles approach one another. An alternative situation in which the current sheet forms is after an instantaneous change in the dipole separation distance from $2a_0$ to $2a$; information would propagate up from the source to N over the Alfvén travel time and the configuration of figure 2 would remain until Ohmic dissipation changes the structure of the sheet. (Axford (1967) has suggested that steady state reconnection can occur at any rate whatever, with the dimensions of a diffusion region adjusting themselves accordingly. This would imply, if such a steady state is set up sufficiently rapidly, that no current sheet forms in the above configuration. However, Axford's hypothesis would be false if there were some lower limit on the size of the diffusion region (Sonnerup 1972) or if the conditions mentioned by Priest & Cowley (1975) were to hold.

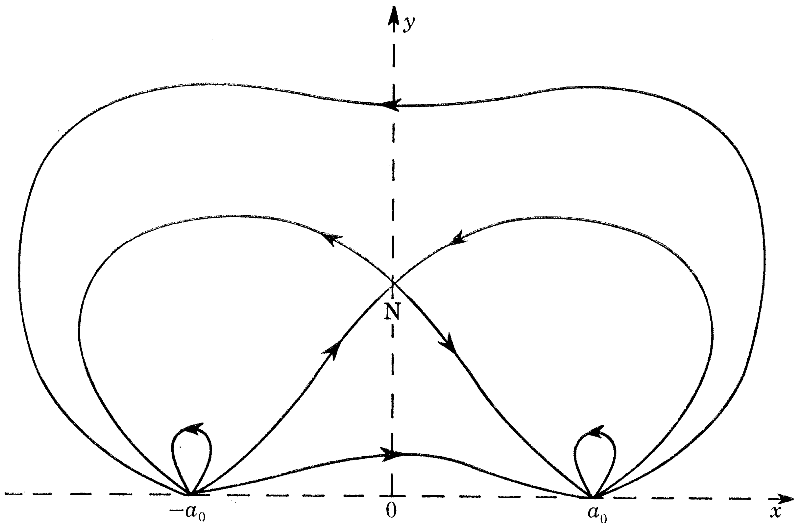

FIGURE 1. The potential magnetic field configuration in a plane perpendicular to two line dipoles of equal strength a distance $2a_0$ apart. (From Priest & Raadu 1975.)

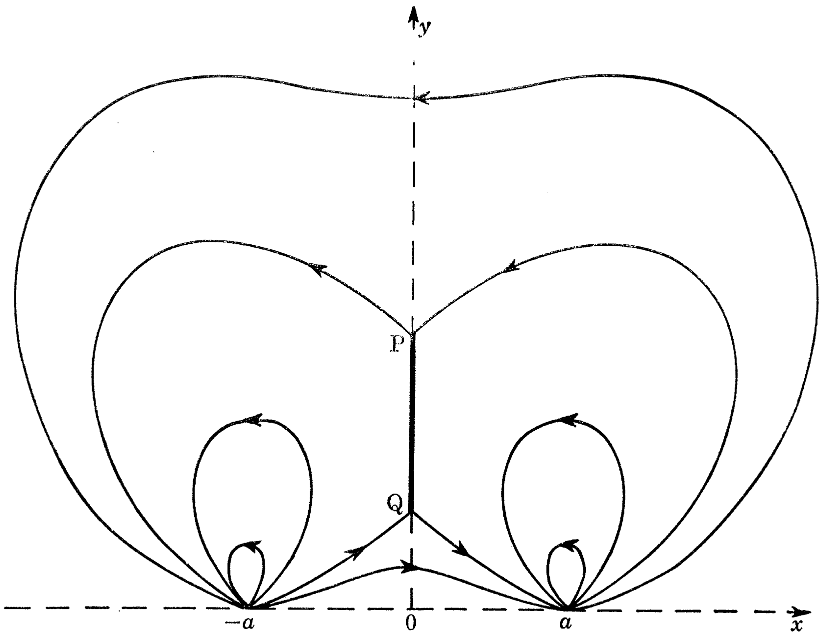

FIGURE 2. The magnetic field when the dipoles are a distance $2a$ ($< 2a_0$) apart and no reconnection has been allowed so that a current sheet stretches from P to Q. (From Priest and Raadu 1975.)

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Furthermore, it is as yet unclear what rate is given by a fully consistent collisionless analysis (Cowley, 1973), which is more relevant to the corona than a fluid model.)

Outside the sheet and the dipoles, the magnetic field is current-free and so potential; its components in the combination $(B_y + \mathrm{i}B_x)$ must therefore be an analytic function of z. In other words we require a function which is analytic in the upper-half z-plane with a cut from P to Q and which behaves like $(z^2 - a^2)^{-2}$ near $\pm a$. The one which possesses these properties is

$$B_y + \mathrm{i}B_x = \frac{E(z^2+p^2)^{\frac{1}{2}}(z^2+q^2)^{\frac{1}{2}}}{(z^2-a^2)^2}, \tag{2}$$

where $E = 4\,\mathrm{i}Da^2\,(a^2+p^2)^{-\frac{1}{2}}(a^2+q^2)^{-\frac{1}{2}}$.

The values of p and q can be found in terms of a/a_0 from the two conditions of frozen-in flux as follows. First of all, the total flux crossing the positive y-axis is

$$\phi = \int_0^\infty (B_x)_{x=0}\,\mathrm{d}y,$$

or, by using equation (2) and the fact that the integrand is an even function of y,

$$\phi = \tfrac{1}{2}E\,Re\int_{-\infty}^{\infty}(p^2-y^2)^{\frac{1}{2}}(q^2-y^2)^{\frac{1}{2}}(a^2+y^2)^{-2}\,\mathrm{d}y.$$

The integral may be evaluated in the standard way by completing the contour with a semi-circle in the upper half plane and evaluating the residue at $\mathrm{i}a$ to give

$$\phi = \tfrac{1}{4}\pi E(p^2q^2-a^4)\,(p^2+a^2)^{-\frac{1}{2}}\,(q^2+a^2)^{-\frac{1}{2}}\,a^{-3}. \tag{3}$$

Initially, $p = q = a = a_0$ and so $\phi = 0$. The assumption that there is no reconnection of field lines implies that ϕ remains zero as the dipoles approach, so that we have from equation (3)

$$pq = a^2. \tag{4}$$

Secondly, the fluxes across ON and OQ must be equal. In other words, integrating the values of B_{0x} and B_x, taken from equations (1) and (2), along the y-axis,

$$D/a_0 = E\int_0^q (p^2-y^2)^{\frac{1}{2}}\,(q^2-y^2)^{\frac{1}{2}}\,(a^2+y^2)^{-2}\mathrm{d}y, \tag{5}$$

which may be evaluated numerically. Thus equations (4) and (5) determine p and q, the result being shown in figure 3.

It should be noted that this configuration is being considered not because it represents a realistic magnetic field but because it is a simple one by means of which current sheet behaviour may be studied. Indeed, the analysis has been extended by Tur (1976) to include the interaction of unequal dipoles, the potential magnetic field lines of which are shown in figure 4. Such a configuration provides a simple model for the emergence of new magnetic flux near the edge of an active region. Tur supposes that the positions of the dipoles remain fixed but the smaller one increases in strength. The resulting current sheet is curved; a sequence of its positions is shown in figure 5 for the case when no reconnection is allowed and the ratio of the dipole moments increases from an initial value of 0.1 to a final value of unity.

The configurations with current sheets contain more magnetic energy than the corresponding ones with the same source positions but purely potential fields. The possibility therefore arises of releasing the excess energy in the form mainly of heat as a solar flare (or in the form mainly of kinetic energy as a surge) without altering the normal component of field at the photosphere (taken to be the x-axis). The amount of energy stored in the above current sheet configurations has been estimated (Priest & Raadu 1975; Tur 1976), but it is by no means clear what triggers the release of the energy. One possibility is that the sheet becomes unstable if its

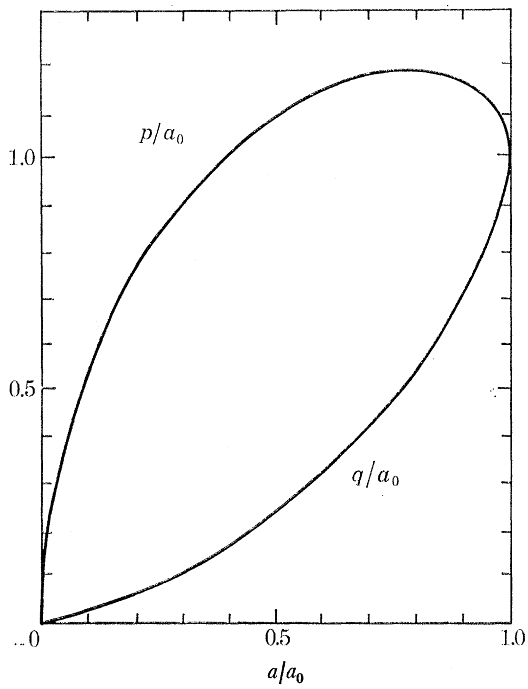

FIGURE 3. The variation of the heights p/a_0 and q/a_0 of the top and bottom of the neutral sheet as a function of a/a_0 as it decreases from unity to zero. (From Priest & Raadu 1975.)

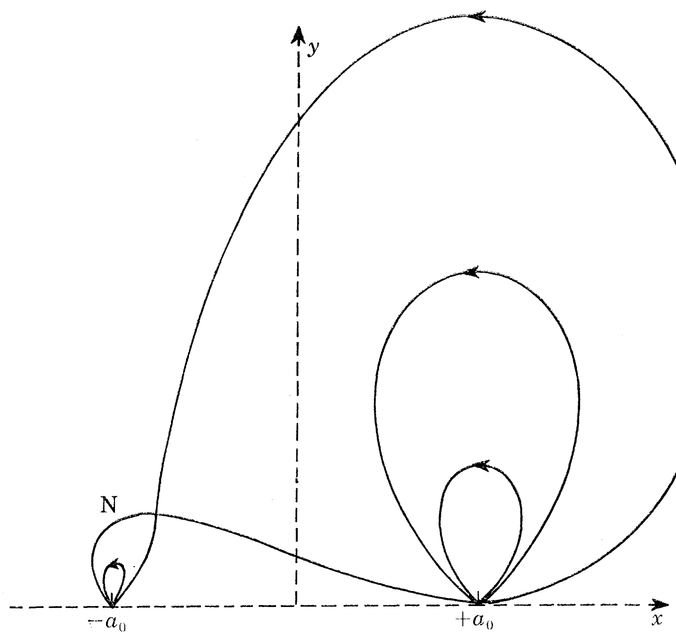

FIGURE 4. The potential magnetic field configuration in a plane perpendicular to two line dipoles with different dipole moments a distance $2a_0$ apart.

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curvature becomes too large. Another is that, at some critical stage, rapid magnetic field annihilation is triggered by a sudden increase in the electrical resistivity of the current sheet due to one of two effects. Either the current density becomes so large that current-induced micro-instabilities set in and lead to a turbulent resistivity, as in the magnetic field reconnection experiment of Baum & Bratenahl (1974). Alternatively, the current sheet may become so long that a local thermal instability lowers the temperature and so increases the value of the Spitzer resistivity; the temperature decrease would occur only in the central diffusion region of a Petschek type of process (Petschek 1964), thus still allowing considerable heating in the surrounding shock waves. However, a trigger may be unnecessary if the current sheet forms at such a height that sufficiently rapid annihilation occurs when ohmic diffusion based on the Spitzer resistivity has had time to act. These ideas and a magnetic configuration which agrees more closely with solar flare observations have been briefly presented by Canfield, Priest & Rust (1974) and developed as a new solar flare model by Heyvaerts, Priest & Rust (1976).

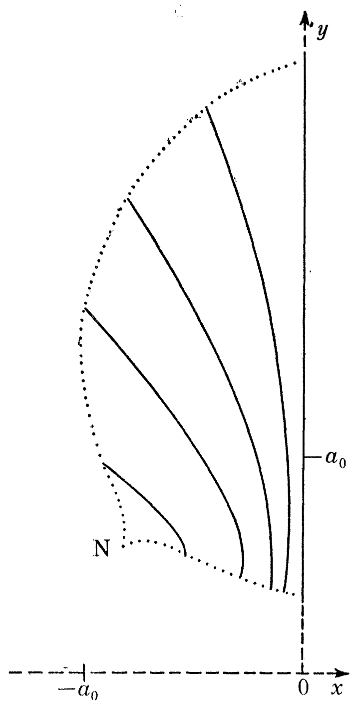

FIGURE 5. A sequence of positions of the current sheet which forms as one dipole at $-a_0$ on the x-axis increases in strength while another dipole at $+a_0$ remains constant. The ratio of the strengths increases from a tenth, when there is no sheet and a simple neutral point at N, to unity when the sheet is straight. The envelope of the two ends of the sheet is shown dotted. (From Tur 1976.)

3. Thermal instability

Thermal instabilities in current sheets may be of relevance to the formation of solar prominences (Kuperus & Tandberg-Hanssen 1967), the triggering of flares in the manner mentioned above and the occurrence of 'neutral line absorbing features' (Priest & Heyvaerts 1974), which are possibly surges seen on the disk.

It is well known that the upper levels of the solar atmosphere have a tendency to be thermally unstable due to the form of the radiative loss term in the energy equation (Parker 1953; Field 1965). Suppose that the equilibrium is characterized most simply by a balance between a constant mechanical heating, G, due to wave dissipation and a radiative, loss $a\rho_0$, proportional to the density, where a is a constant:

$$0 = G - a\rho_0.$$

A perturbation away from the equilibrium temperature T_0 will satisfy the energy equation

$$c_p \frac{\partial T}{\partial t} = G - a\rho, \tag{6}$$

which, on using the equilibrium equation and the perfect gas law with the pressure assumed constant, becomes

$$c_p \frac{\partial T}{\partial t} = b\left(\frac{1}{T_0} - \frac{1}{T}\right),$$

where b is a positive constant. Thus, if $T < T_0$, $\partial T/\partial t$ is negative and the perturbation continues. The cooling time for this thermal instability is given by equating the orders of magnitude of both sides of equation (1) as

$$\tau_c = c_p\, T_0/(a\rho_0).$$

For a temperature of 2×10^5 K and a density characteristic of the high chromosphere, one finds $\tau_c \sim 100$ s, in agreement with observations of absorbing features. For coronal values, on the other hand, one derives a cooling time somewhat greater than the day which is needed for prominence formation.

In most of the solar atmosphere any temperature perturbation is wiped out by the effect of thermal conduction so that the thermal instability is suppressed. Inside a current sheet, however, the plasma is to a certain extent thermally insulated by the magnetic field, since heat can be conducted effectively along field lines but not normal to them. Thus, if the sheet is short enough the instability is still suppressed, but, when the sheet length exceeds a certain maximum value, the instability can occur. The characteristic time for conducting heat along a length L of the magnetic field is

$$\tau_{\parallel} = L^2/\kappa_{\parallel},$$

where $\kappa_{\parallel}$ is the coefficient of thermal conductivity parallel to the field. If L is so small that $\tau_{\parallel} < \tau_c$ then the plasma is thermally stable. The maximum value of L for which this is so comes from equating $\tau_{\parallel}$ and τ_c; in the upper chromosphere it is about 500 km, whereas in the corona it can reach 50000 km, the former value being typical of absorbing feature dimensions and the latter of prominence heights.

The above order of magnitude analysis has been improved upon recently by Smith (1976). He considers an idealized equilibrium current sheet of length L with zero internal magnetic field and internal pressure, density and temperature denoted by p_2, ρ_2, T_2. If the corresponding values outside the sheet, denoted by subscript 1, are specified, then p_2, ρ_2, T_2 are determined by the pressure balance

$$p_2 = p_1 + B_1^2/(2\mu),$$

the perfect gas law

$$p_2 = R\rho_2 T_2,$$

and the energy equation

$$0 = a\rho_1 - a\rho_2 + \frac{\kappa_{\parallel}(T_2 - T_1)}{L^2 \rho_2},$$

in which the first term on the right represents the constant mechanical heating, assumed to balance the external radiative loss, and the last term is an order of magnitude estimate for the conduction term, $\kappa_{\parallel}$ being proportional to $T_2^{\frac{5}{2}}$. p_2 and ρ_2 may be eliminated between these three equations to give T_2 as a function of L, the form of which is shown in figure 6. Starting from the point on the curve where $T_2/T_1 = 1$ when $L = 0$, the equilibrium internal temperature T_2 decreases with increasing L up to the value L_{max}, beyond which no equilibrium solution is possible. For each value of L less than L_{max} there is an alternative equilibrium solution for T_2 to the left of the maximum; it turns out, however, to be thermally unstable, whereas the solution to the right of the maximum is stable. These results are modified if the current sheet is so narrow and the magnetic field so large that Joule heating and heat conduction across field lines need to be included in the energy equation.

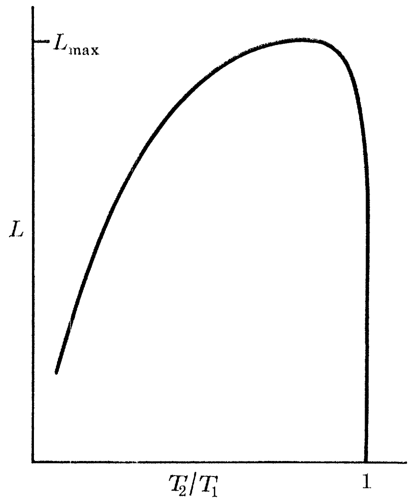

FIGURE 6. The length L of a current sheet in thermal equilibrium as a function of the ratio of the internal temperature T_2 to the external temperature T_1.

4. STEADY MAGNETIC FIELD ANNIHILATION

A current sheet tends to broaden in time due to the effect of finite electrical conductivity. However, a steady state may be maintained if this broadening is balanced by an inflow of magnetic flux and plasma from the sides. Recently, by extending some work of Parker (1973), Priest & Sonnerup (1975) discovered an exact solution of the magnetohydrodynamic equations which describes such a process. It has the two dimensional incompressible stagnation point flow

$$v_x = -kx, \quad v_y = ky,$$

where k is constant, together with the unidirectional field

$$B_x = 0, \quad B_y = B_y(x).$$

These automatically satisfy the equations div $\boldsymbol{B}$ = div $\boldsymbol{v}$ = 0 and the form of B_y is determined by Ohm's law,

$$E_z - kxB_y = \eta \frac{dB_y}{dx}, \tag{7}$$

where E_z is a constant and η is the magnetic diffusivity. In addition, since the velocity is curl-free and the field lines straight, the equation of motion reduces to a Bernouilli type of equation which serves to determine the gas pressure everywhere.

The stagnation point flow is shown in figure 7 and brings frozen-in magnetic flux from the sides towards the plane $x = 0$. In the process, the field strength builds up, until one enters the shaded diffusion region, where the current is so large that the right hand side of equation (7) is significant and the field can be annihilated. The resulting solution for the magnetic field is shown in figure 8.

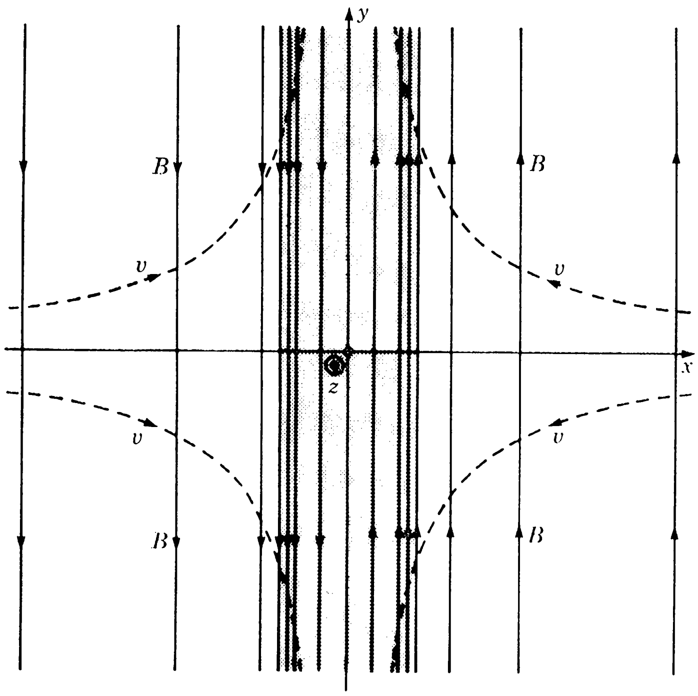

FIGURE 7. Magnetic field lines (solid lines) and streamlines (broken lines), with the diffusion region indicated by shading. (From Priest & Sonnerup 1975.)

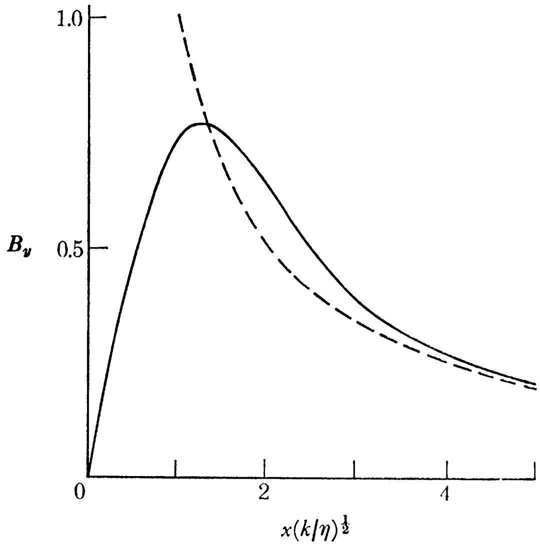

FIGURE 8. The magnetic field, normalized to make its gradient unity at the origin, as a function of distance. Also shown for comparison by a dashed line is the frozen field profile $B_y = E_z/(kx)$.

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There have been several steady state magnetic field annihilation models. The diffusion region for the present one is of infinite extent in the y direction; it is therefore relevant to the case of 'slow' magnetic merging when flux is carried in at a speed significantly less than the Alfvén speed and the diffusion region is much longer than it is wide. Moreover, the main importance of the present model is that it can be generalized to include fields and flows in the z direction. In particular, it is possible to find solutions which join an arbitrary field on one side of the sheet to an arbitrary field on the other side.

5. Conclusion

The relation between the three aspects of current-sheet theory here presented depends on the application in mind. For solar flares, the first aspect may be relevant to the build-up of a current sheet before the flare when little magnetic field annihilation is taking place. In §2 it was demonstrated how to calculate the position and shape of the current sheet which forms in simple configurations and the amount of magnetic energy which is thus stored may also be evaluated. The thermal instability, described in §3, may be relevant to the trigger mechanism which leads to a release of the stored energy; its main application is, however, to the problem of prominence formation. Finally, the actual release of energy in a flare is widely believed to be caused by *rapid* annihilation of magnetic flux. The model of magnetic field annihilation, described in §4, holds only for energy release rates which are smaller than one observes in a flare; its main interest lies in being an exact solution of the magnetohydrodynamic equations and in being able to describe the annihilation of fields which are not simply antiparallel but are arbitrarily inclined.

References (Priest)

Axford, W..I. 1967 *Space Sci. Rev.* **7**, 149.
Baum, P. J. & Bratenahl, A. 1974 *J. Plasma Phys.* **11**, 93.
Canfield, R. C., Priest, E. R. & Rust, D. M. 1974 *Proc. of High Altitude Observatory Seminar on Flare-related magnetic field dynamics*, Boulder, Colorado.
Cowley, S. W. H. 1973 *Cosmic Electrodynamics* **3**, 448.
Field, G. B. 1965 *Astrophys. J.* **142**, 531.
Green, R. M. 1965 *I.A.U. Symp.* no. 22, 398.
Heyvaerts, J., Priest, E. R. & Rust, D. M. 1976 (In preparation.)
Kuperus, M. & Raadu, M. A. 1974 *Astron. Astrophys.* **31**, 189.
Kuperus, M. & Tandberg-Hanssen, E. 1967 *Solar Phys.* **2**, 39.
Parker, E. N. 1953 *Astrophys. J.* **117**, 431.
Parker, E. N. 1973 *J. Plasma Phys.* **9**, 49.
Petschek, H. E. 1964 AAS-NASA Symp. *Physics of solar flares* (NASA SP-50, p. 425).
Priest, E. R. & Cowley, S. W. H. 1975 *J. Plasma Phys.* **14**, 271.
Priest, E. R. & Heyvaerts, J. 1974 *Solar Phys.* **36**, 433.
Priest, E. R. & Raadu, M. A. 1975 *Solar Phys.* **43**, 177.
Priest, E. R. & Sonnerup, B. U. Ö. 1975 *Geophys. J. R. Astron. Soc.* **41**, 405.
Raadu, M. A. & Kuperus, M. 1973 *Solar Phys.* **28**, 77.
Rust, D. M. 1976 *Phil. Trans. R. Soc. Lond. A* **281**, 427–443 (this volume).
Smith, E. A. 1976 Ph.D. thesis, St Andrews University. (In preparation.)
Sonnerup, B. U. Ö. 1972 *NASA Symp. on High energy phenomena on the Sun*, Greenbelt, Maryland.
Sweet, P. A. 1958 *I.A.U. Symp.* no. 6, p. 123.
Tur, T. 1976 Ph.D. thesis, St Andrews University. (In preparation.)
Uchida, Y. 1969 *Pub. Astron. Soc. Japan* **21**, 128.
Zirin, H. 1974 *Vistas Astron.* **16**, 1.